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ABSTRACT

Paper presented at a regional convention of the National Council of Teachers of Mathematics, Atlanta, Georgia, November, 1971. Described is some of the recent literature in classroom communication analysis based on fundamental cognitive processes. A logical analysis of teaching and teacher verbal behavior is presented. Included is a description of the strategies of teaching involved in teaching concepts, generalizations, and skills in mathematics. The subsequent implications for training teachers of mathematics are discussed including applications to microteaching. (JG)

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A Basis For Training Mathematics Teachers

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by

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One of the primary concerns of mathematics educators is the preparation of prospective teachers. The problem of preparing teachers is complex and is complicated by the fact that good teaching is an elusive and ill-defined concept. Presently, efforts to train mathematics teachers have been diverse and generally not based on any theoretical foundation.[1] It is the intent of this article to explore a basis for developing a teacher training program, a basis which has the beginnings of a theoretical structure on which to build.

We do know some things about teaching. Flanders has provided us with procedures for measuring a particular aspect of teaching and relating this component to student achievement.[2] We also know some things are not characteristic of good teaching. For example, students generally seem to comprehend less when exposed to an increasing number of major ideas in a lecture. We know that certain modes of teaching, e.g., teaching by discovery, are effective in some contexts and not effective in other contexts.

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[1]The use of the term 'train' in education may be inappropriate. If one has trained someone to do something it is assumed that the trainee then performs in a certain and specified way. However, in education when we speak of training teachers we do not use 'train' in the same sense. Because teaching is basically an art, at least presently, it is not possible to train a teacher in the usual sense of the word.

[2]Flanders, Ned. "Some Relationships Among Teacher Influence, Pupil Attitudes, and Achievement," Contemporary Research on Teacher Effectiveness, ed. Biddle, Bruce and Ellena, William. New York: Holt, Rinehart, and Winston, 1964, pp. 196-231.

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It seems, then, that teaching is sort of a "quasi-art." That is, although some influencing factors are known about teaching it is still largely a matter of individual artistry that determines whether the teaching is successful. Robert Davis suggests that our goal in teacher education can best be accomplished by having prospective teachers imitate master teachers. The basis of his position is that teaching is an art and hence the optimum strategy to develop good teachers is to allow them to practice their artistry under the guidance of a superior artist teacher.[3]

The extent to which teaching can be meaningfully subjected to an objective analysis is an open question. Nevertheless it seems reasonable to assume that the development of superior teachers can be enhanced by providing the trainees with models and strategies for teaching the subject matter at hand. This is not to say that such models or strategies need to be couched in prescriptive language. Rather they provide a logical basis for the teacher to choose various strategies of teaching he deems important. It seems more profitable not to regard teaching as solely an art and hence limit teacher training to the imitative of master teachers, but rather to provide teachers with what scientific knowledge can be gathered about teaching in order that teachers can better develop and evaluate their strategies. Let us consider, then, a procedure for training teachers which involves an interplay of logical analysis of teaching and the individual artistry of which Robert Davis speaks.

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[3]Davis, Robert B. "Mathematics Teaching--With Special Reference to Epistemological Problems." Journal of Research and Development in Education. Monograph No. 1, Fall, 1967. pp. 39-43.

If one observes the various types of verbal behavior that teachers utilize, it becomes apparent that a language apart from that of the subject matter is required to describe the teacher's activities. Smith states:

> To be able to talk about the content of instruction is to climb above it and to analyze it from this higher and more inclusive perspective. To do that requires an appropriate level of language and one that the teacher is able to converse in.[4]

Smith argues that teachers who possess this kind of knowledge have an added advantage in analyzing and hence controlling their own teaching behavior. One of the values of such knowledge is that it provides teachers with a foundation for choosing effective teaching strategies.

Whatever else that mathematics teachers do, they surely teach concepts, generalizations, and skills. Let us consider the nature of these types of knowledge, investigate what logical moves can be made in teaching these types of knowledge, and translate this information into a program for training teachers.

Teaching Concepts

Consider the teaching of concepts. If a concept is regarded as a category or mentalistic container denoted by some term. then there are a number of logical ways called 'moves' that information can be conveyed about the element of that set. Given below are moves that have been identified in teaching concepts.[5]

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[4]Smith, B. Othanel. Teachers for the Real World. Washington, D.C. The American Association of Colleges for Teacher Education, 1969, p. 126.

[5]Smith, B. Othanel, et al. A Study of the Strategies of Teaching. Cooperative Research Project No. 1640, U.S. Office of Education, Urbana, Illinois: Bureau of Educational Research, College of Education, University of Illinois, 1964, pp. 58-95.

A. Define the term denoting the set.

B. Give a superset of the set.

C. Give subsets of the set.

D. Give characteristics of objects in the set.

E. Give sufficient conditions for an object to be in the set.

F. Compare and contrast objects either solely within the set or objects within and without the set.

G. Give examples of objects in the set.

H. Give nonexamples.

I. Give counterexamples to correct misconceptions about the concept.

These moves can provide a basis for students to construct and analyze lessons. It is not a question of constructing the optimum strategy but rather it is a question of providing the students with ways of developing alternative strategies for teaching various concepts.

For example, the concept of periodic functions could be taught by showing that periodic functions are special kinds of functions. Characteristics of periodic functions would be discussed along with those factors that differentiate periodic functions from other kinds of functions. These moves can be readily identified with those given above. It should be noted that these moves need not be confined to a lecture - discussion classroom setting. They could involve graphing, the investigation of solution sets of equations involving periodic functions or other activities encompassing these logical moves. An alternative strategy might involve defining 'periodic function' and then finding examples and nonexamples of periodic functions. From these 2 distinct sets the students can investigate what conditions are necessary for a function to be periodic. These moves can be utilized in many different modes of communication.

Discovery techniques can be utilized in teaching concepts by presenting to students a series of examples and nonexamples of the concept. When the teacher feels the students have abstracted the defining attributes of the concept he may ask someone to define the concept. Should the student's definition not include the necessary and sufficient conditions for an object to be in the referent set of the concept, the teacher may use a counterexample to correct any misconceptions. A mathematics teacher could utilize this strategy in teaching the concept of prime number. He might present students with examples and nonexamples of prime numbers, e.g., 12, 7, 10, 11, and 9 and ask the students to form rectangular arrays of dots representing the given numer. The student realizes, hopefully, that the only rectangular arrays that can be made to represent 7 and 11 are 1 x 7 and 1 x 11 respectively. That is, the student abstracts that property which is common to prime numbers, viz., that the only positive factors of a prime number are 1 and the number itself. Should the students incorrectly conclude that prime numbers are necessarily odd, the teacher could give 2 as a counterexample.

Teaching Generalizations

Generalizations are a second kind of knowledge that is taught. A generalization is a statement about a set of objects. The following broad categories have been identified as moves in teaching generalizations:[6]

1. Motivation
2. Clarification
3. Justification
4. Application

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[6]Henderson, Kenneth B. Teaching Secondary School Mathematics. Washington: National Education Association, 1969.

Teachers can provide motivation (from a cognitive viewpoint) for learning a generalization by discussing how the generalization can play a role in the formulation of other generalizations, how this generalization can help solve problems previously unsolvable for students, or by pointing out how the generalization provides a structure to previously learned concepts. A generalization can be clarified by giving instances, making sure the students understand the terminology involved, or by pointing out the hypothesis and conclusion of the generalization. The strategy involved in justifying a generalization is determined to some extent by the nature of the generalization. The truth of a generalization may be predicated upon empirical evidence gathered from various observations. However, mathematics by its very nature provides generalizations whose truth can be determined on the basis of previously accepted definitions and postulates.

Consider, now, how this knowledge can be applied in teaching generalizations. A too often forgotten aspect of teaching is giving students reasons for learning a particular principle. Students should be made aware that the congruence theorems play an important part in developing other theorems used in geometry. The quadratic formula can help students solve equations which were previously unsolvable for them. Lagrange's theorem provides students with information about groups and their subgroups, a fact which students may not have previously recognized. The methods by which reasons for learning generalizations can be communicated to students can vary from straight lecture to challenging problems which are unsolvable for the student with the knowledge he has at hand.

Ausubel's noted statement:

> The most important single factor influencing learning is what the learner already knows. Ascertain this and teach him accordingly.[7]

has particular relevance in clarifying generalizations. In order for an item of knowledge to have meaning for a student, he must be able to assimulate this knowledge into his existing cognitive structure. Hence, part of a teacher's job is to determine what knowledge the student has at hand in order to relate the new knowledge to his present knowledge. In teaching the theorem:

> The centroid of a triangle divides the medians into a ratio of 2 to 1.

the teacher must ascertain whether the students have a sound grasp of the relevant concepts, e.g., centroid, median, and ratio. Surely if the student does not thoroughly understand these concepts, the principle can not have real meaning to him. The generalization that "any even number greater than 4 can be expressed as the sum of 2 odd primes." (Goldbach's conjecture) can be clarified by giving instances of the generalization, e.g., $8 = 5 + 3$, $12 = 7 + 5$, and $30 = 19 + 11$.

A generalization can be justified in a number of different ways. The basic moves involve giving supporting instances, searching and not finding counterinstances, giving deductive proofs or showing that the generalization follows from a subsuming generalization. Combinations of these moves could be used in constructing various strategies. For example, Goldbach's conjecture can be justified, or at least made more believable, by generating a sequence of instances and by trying in vain to find a

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[7]Ausbel, David P. Educational Psychology, A Cognitive View. Chicago: Holt, Rinehart, and Winston, Inc., 1968, frontispiece.

counterinstance. Other mathematical principles, such as the Pythagorean Theorem, can be proven deductively.

Teaching Skills

Another important aspect of teaching is the teaching of skills. Skills are a different type of knowledge than generalizations and concepts. Skills imply speed and accuracy. While it makes sense to say he can solve quadratic equations with great speed and accuracy, it seems odd to say he knows the quadratic formula with great speed and accuracy.

By and large skills are taught through the use of directives or prescriptions. Prescriptions serve the purpose of providing a link between generalizations and observable activities. While prescriptions are not truth functional they can be derived from generalizations which are truth functional. This suggests a strategy in the teaching of skills.

Since skills can often be learned by imitation, it makes sense to demonstrate the skills to the student and then allow the students to practice their expertise in learning the skill. The field of behavioral psychology can provide a number of ideas to be considered in teaching a skill, such as the concept of feedback and the role of practice. However,the teacher may also help the student learn in a meaningful way the generalization on which the prescription for the skill is based. This would seem to have particular benefit for retention purposes.

Training Mathematics Teachers

To have students acquire the knowledge discussed in the preceeding paragraphs could be a first step in a teacher training program. However, the real essence of the matter is to allow students to utilize this knowledge in developing their own artistry of teaching. Microteaching provides a vehicle for students to accomplish this end. Through the use of

microteaching sessions, students have not only the opportunity to create and teach new strategies but also an opportunity to evaluate and revise other strategies.

As research is completed on the effectiveness of various strategies, teachers can have a basis for selecting teaching techniques which are based on more than a common sense notion of effective teaching. In any event, the utilization of logical analyses of teaching and the use of theoretical models can provide prospective teachers with a basis for developing their own practitioner's maxim.

The Matter of Accountability

One of the growing ideas in education today is the notion of accountability. The idea is quite simple - one is evaluated on the basis of the product he produces.

Consider for the moment the concept of accountability as applied to undergraduate teacher education programs. While it may not be in the best interests of education to place undergraduate programs under the domain of accountability, at least in its strictest sense, the notion of accountability could no doubt stimulate some rather innovative changes in current programs.

It seems rather strange that the crowning glory of most teacher education programs, student teaching, generally occurs in a situation in which the institution granting the degree has little jurisdiction save for a few visits by a college supervisor who is often a graduate student. We strive to have students take mathematics, psychology, history, and philosophy courses that educators deem necessary to be an effective teacher. We can with a fair degree of accuracy determine whether or not the student has learned this knowledge.

But what level of confidence do we have that when a student graduates he can teach mathematical concepts, generalizations or skills effectively? I suggest we actually have little confidence, namely, because we haven't trained him to do so. The discussion above can serve as a foundation for developing a competency based program which is both theoretically sound and practically useful. Such a program could provide information about a student's ability to construct and effectively carry out a variety of teaching strategies and at the same time incorporate findings that arise from the research literature.

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